Math 10A
Worksheet, Discussion #29; Thursday, 7/26/2018
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1 Euler's Method

1.1 Concepts

1. Euler's method allows us to approximate solutions to differential equations. Given a differential equation $y' = f(y, t)$ and an initial condition $y(0) = y_0$ and a step size h, we can approximate the path by $y_{n+1} = y_n + f(y_n, t_n)h$. This is gotten by writing $y' = \frac{dy}{dt} \approx \frac{y_{n+1} - y_n}{h}$.

   A slope field is a graph where at every point y, t, you draw a line with the slope there, which is given by the function $f(y, t)$.

1.2 Problems

2. **TRUE** False We can only use slope fields and Euler's method when we are given a first order equation.

3. Consider the differential equation $y' = x - y^2$ with initial condition $y(0) = 1$. Use Euler's method to approximate $y(3)$ using step sizes of 1.

   Solution: We use the fact that $y' = \frac{dy}{dx} \approx \frac{y(1)-y(0)}{1-0} = 0 - y(0)^2 = -1 = \frac{y(1)-y(0)}{1}$ and hence $y(1) \approx y(0) - 1$. Alternatively, we can write $y(n+1) \approx y(n) + f(n, y(n))(1) = y(n) + n - y(n)^2$. Thus $y(1) \approx 1 + 0 - 1 = 0$ and $y(2) \approx 0 + (1 - 0^2) = 1$ and $y(3) \approx 1 + 2 - 1^2 = 2$.

4. Use Euler's method to estimate $y(3)$ given that $y' = x^2 + y^2$ and $y(0) = 0$ using step sizes of 1.

   Solution: We have $y(1) \approx y(0) + (0^2 + 0^2)(1) = 0$ and $y(2) \approx y(1) + (1^2 + 0^2)(1) = 1$ and $y(3) \approx y(2) + (2^2 + 1^2)(1) = 1 + 5 = 6$.

5. Use Euler's method to estimate $y(3)$ given that $y' = y^2 - x^2$ and $y(0) = 1$ using step sizes of 1.

Solution: We have $y(1) \approx y(0) + (1^2 - 0^2)(1) = 2$ and $y(2) \approx y(1) + (2^2 - 1^2)(1) = 2 + 3 = 5$ and $y(3) \approx y(2) + (5^2 - 2^2)(1) = 5 + 21 = 26$.

2 Slope Fields

2.1 Concepts

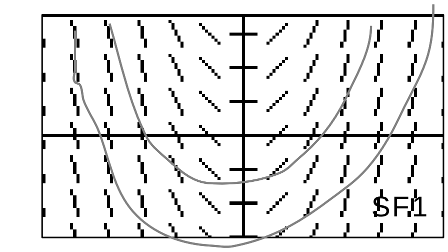
6. A slope field is a graph where at every point y, t, you draw a line with the slope there, which is given by the function $f(y, t)$.

2.2 Problems

7. **TRUE** False Autonomous equations like $y' = 2\sqrt{y}$ will have slope field that are the same after shifting left and right.
8. Match each slope field to the differential equation and sketch some solutions to them.
9. Draw a slope field for $y' = y^2 + x^2$ and sketch the solution when $y(0) = 0$ on the interval $-2 \le x \le 2, -2 \le y \le 2$.
10. Draw a slope field for $y' = y^2 - x^2$ and sketch the solution when $y(0) = 1$ on the interval $0 \le x \le 4, 0 \le y \le 4$.
11. For each differential equation, estimate $y(2)$ using the starting point $y(1) = 1$ and step size of $h = \frac{1}{2}$.

Solution:

DE	$y(1.5)$	$y(2)$
	$y(1) + f(1, y(1))h$	$y(1.5) + f(1.5, y(1.5))h$
1	$1 + (1-1)(0.5) = 1$	$1 + (1.5-1)(0.5) = 1.25$
2	$1 + (1/1)(0.5) = 1.5$	$1.5 + (1.5/1.5)(0.5) = 2$
3	$1 + (1-1)(0.5) = 1$	$1 + (1-1.5)(0.5) = 0.75$
4	$1 + (-1/1)(0.5) = 0.5$	$0.5 + (-1.5/0.5)(0.5) = -1$
5	$1 + (1)(0.5) = 1.5$	$1.5 + (1.5)(0.5) = 2.25$
6	$1 + (-1/1)(0.5) = 0.5$	$0.5 + (-0.5/1.5)(0.5) = \frac{1}{3}$
7	$1 + (1/2)(0.5) = 1.25$	$1.25 + (1.25/2)(0.5) = \frac{25}{16}$
8	$1 + 0.25(1)(4-1)(0.5) = 1.75$	$1.75 + 0.25(1.75)(2.25) = \frac{175}{64}$
9	$1 + (2-1)(0.5) = 1.5$	$1.5 + (2-1.5)(0.5) = 1.75$
10	$1 + (1+1)(0.5) = 2$	$2 + (1.5+2)(0.5) = 3.75$



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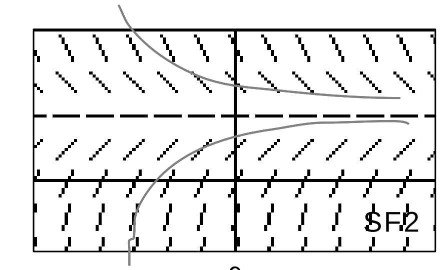

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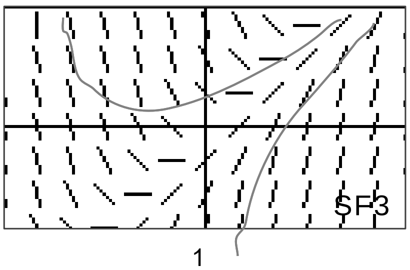

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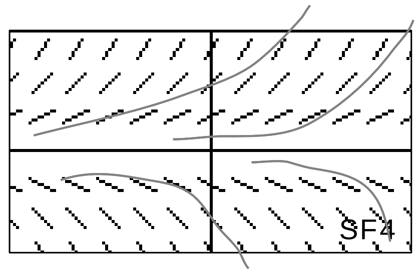

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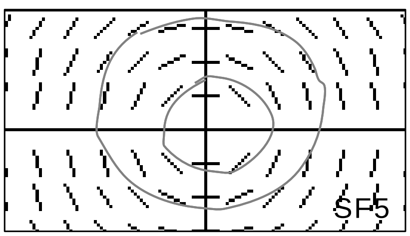

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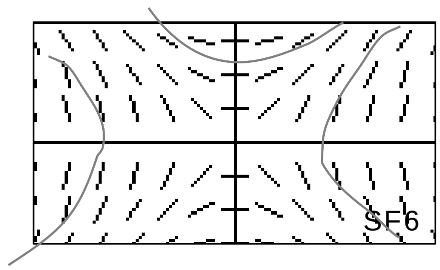

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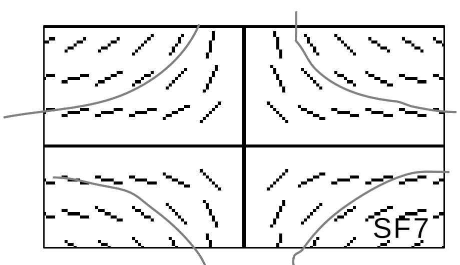

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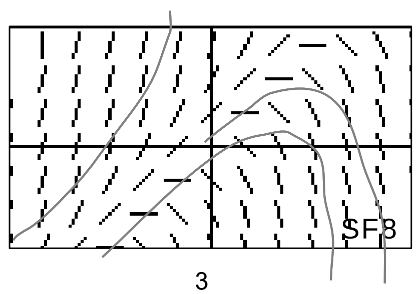

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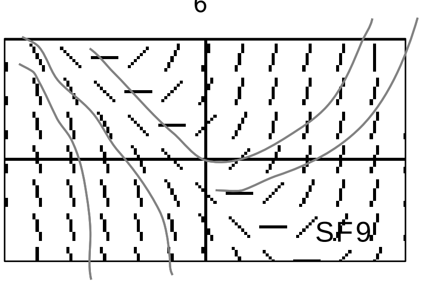

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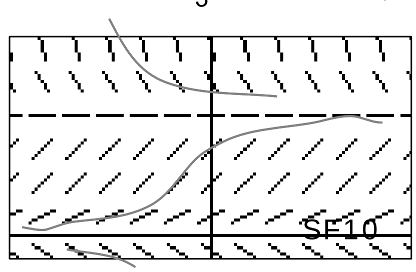

8

$$\frac{dy}{dx} = x - y$$

DE1

$$\frac{dy}{dx} = \frac{x}{y}$$

DE2

$$\frac{dy}{dx} = y - x$$

DE3

$$\frac{dy}{dx} = -\frac{x}{y}$$

DE4

$$\frac{dy}{dx} = x$$

DE5

$$\frac{dy}{dx} = -\frac{y}{x}$$

DE6

$$\frac{dy}{dx} = \frac{y}{2}$$

DE7

$$\frac{dy}{dx} = 0.25y(4 - y)$$

DE8

$$\frac{dy}{dx} = 2 - y$$

DE9

$$\frac{dy}{dx} = x + y$$

DE10